Structure of Turbulent Jets and Wakes

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The structure of two-dimensional turbulent jets and wakes is studied using two eddy viscosity models. The turbulent energy equation is used together with the mean momentum equations, and the system is closed by introducing eddy coefficients. The fine structure turbulence is obtained by applying proper boundary conditions at the mean turbulent interface position. The results, after a second averaging process, are compared with available experimental data.

Nomenclature

A	= constant coefficient
D	= rate of strain tensor for mean flow
E, H, W	= nondimensional velocity components
K	= universal constant
L	= mixing length
N	= nondimensional eddy viscosity
τ	= Reynolds stress tensor
P	= probability density function
Q	= nondimensional turbulent energy flux
T	= characteristic time scale
U	= turbulent intensity
V	= entrainment velocity at turbulent interface
h	= turbulent interface position
$\bar{h}$	= mean turbulent interface position
n	= mixing length ratio
$\bar{p}$	= mean static pressure
$\mathbf{q}$	= turbulent energy flux vector
$\mathbf{u}$	= mean velocity vector
u, v	= velocity components
$\tilde{u}$	= defect velocity
α	= universal constant
γ	= intermittency factor
ε	= viscous dissipation rate
σ	= standard deviation
ξ, η	= similarity variables
ν	= kinematic viscosity
ν_T	= eddy viscosity
Θ	= nondimensional turbulent energy

Subscripts

∞	= freestream condition
j	= jump condition
$\mathbb{C}$	= centerline condition

I. Introduction

TURBULENT shear flows can be divided into wall turbulence and free turbulence. In wall turbulence, the flow is characterized by the existence of a thin wall layer where the majority of the turbulent energy production in the flow occurs. The excess turbulent energy produced in the wall layer is then diffused away to the bulk of the flow where the viscous dissipation is at work. This continuous transfer of turbulent energy across the flow is typical of wall turbulence, and can be found in turbulent flows in pipes, channels, and boundary layers.[1-4]

In free turbulence, a completely different structure prevails. Townsend[5] in his study of the turbulent wake found that the distribution of turbulent energy and dissipation are nearly uniform in the main body of the flow, and the transfer of turbulence is no longer dominated by gradient diffusion as in wall turbulence. Instead, a large-scale convection exists which transfers turbulence in bulk. This large-scale motion is also observed in turbulent flows in jets,[6,7] mixing layers,[8] and the outer portion of boundary layers.[4]

A model with double structure was proposed by Townsend[9] in which a group of large eddies is superimposed on the fine structure of turbulence. The large eddies are an order of magnitude greater than the energy carrying eddies, and they transfer turbulent energy convectively. The large eddy motion distorts the turbulent interface, leading to the well observed phenomenon of intermittency. Under the concept of this model two different modes of turbulent transport exist, which can be treated separately.

Two eddy viscosity models for free turbulent flows were studied in a previous paper.[10] In these models, the turbulent energy equation is used with a gradient diffusion term for turbulent energy flux. The turbulent diffusivity is assumed to be proportional to the eddy viscosity. By applying proper boundary conditions at a smooth turbulent interface, the fine structure of turbulence can be obtained, which is somewhat similar to the conditional averages of Kovasznay et al.[11] Conventional averages, which are measured irrespective of the position of the turbulent interface, can then be obtained by performing a second averaging using the turbulent interface position as the random variable, similar to the treatment of Nee and Kovasznay.[12]

The distribution of the turbulent interface position can be related to the intermittency factor γ which is defined as the probability that the flow is turbulent at a given point. Assuming the interface position to be a single valued function of the coordinates, the probability density distributions of the interface position is given by the derivative of the intermittency factor. From measurements in many flows, it was found that the distribution of γ can be approximated by error curves,[13] giving the following Gaussian distribution for the probability density function

$$P(h) = \frac{1}{\sigma}\exp\left[-\frac{1}{2}\left(\frac{h-\bar{h}}{\sigma}\right)^2\right] \tag{1}$$

Here h is the instantaneous position of the interface, $\bar{h}$ its mean, and σ the standard deviation. Assuming that h is at the position

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Table 1 Values of standard deviation of interface position

Type of flow	$\sigma/\bar{h}$
Turbulent plane wake	0.35
Turbulent plane jet	0.216
Outer portion of turbulent boundary layer	0.18

where $\gamma = 0.5$, the value of $\sigma/\bar{h}$ can be obtained experimentally. A list of these values is given in Table 1.

To perform the second averaging, it is necessary to know the response of the turbulent structure to the fluctuation in the turbulent interface. Guided by similarity arguments, a simple, linear response is used in this paper. Assuming a similarity solution for the fine structure given by $F(\xi)$ where $\xi = y/\bar{h}$, the solution $f(y, t)$ of the same flow quantity at time t and position y for a problem with fluctuating interface can be given by a "breathing law"[12]

$$f(y, t) = F(z) \tag{2}$$

where

$$z = \begin{cases} (y - h + \bar{h})/\bar{h} & y > h \\ y/h & y \leq h \end{cases} \tag{3}$$

and h is the instantaneous position of the interface.

Under the "breathing law," the whole turbulent layer is stretched and compressed linearly with the change in interface position, while outside the turbulent layer the flow is only shifted by a constant value. This assumption for $y > h$ is arbitrary because all flow quantities considered here are assumed to be zero for $y > h$, making no difference in what is used there.

The conventional average for f at position y is then given by

$$\bar{f}(y) = \left(\int_0^\infty F(z')P(h)\,dh \Big/ \int_0^\infty P(h)\,dh \right) \tag{4}$$

where $P(h)$ is the probability density function for the interface position, given by Eq. (1). For flow quantities which are characteristic of turbulent flow $F(z) = 0$, for $z > 1$ then Eq. (4) becomes

$$\bar{f}(y) = \left(\int_y^\infty F(y/h)P(h)\,dh \Big/ \int_y^\infty P(h)\,dh \right) \tag{5}$$

In this paper, the structure of turbulent jets and wakes is studied using this approach. The fine structures are obtained first using the eddy viscosity models, and Eq. (5) is then used to give the conventional averages. Results are then compared with available experimental data.

II. Eddy Viscosity Models

The equations for the fine structure of turbulence are given in Ref. 10. Written in vector form, they are

$$(\partial/\partial \mathbf{r}) \cdot \mathbf{u} = 0 \tag{6}$$

$$(D\mathbf{u}/Dt) = -(1/\rho)(\partial/\partial \mathbf{r})\,\bar{p} - (\partial/\partial \mathbf{r}) \cdot \bar{\bar{\tau}} + \nu \nabla^2 \mathbf{u} \tag{7}$$

$$\varepsilon + \frac{3}{2}\frac{DU^2}{Dt} = -\tau_{ij}D_{ij} - \frac{\partial q_i}{\partial x_i} + \nu\left(\nabla^2 \frac{3}{2}U^2 + \frac{\partial}{\partial x_i}\frac{\partial}{\partial x_j}\tau_{ij}\right) \tag{8}$$

where

$$\tau_{ij} = U^2\delta_{ij} - 2\nu_T D_{ij}, \quad D_{ij} = \tfrac{1}{2}[(\partial u_i/\partial x_j) + (\partial u_j/\partial x_i)] \tag{9}$$

and

$$q_i = -(3/2)\nu_T\,\alpha(\partial U^2/\partial x_i) \tag{10}$$

Here $\mathbf{u}$ is the (fine structure) mean velocity, $(3/2)U^2$ is the turbulent energy per unit mass, and $\alpha = 1$ is a universal constant.

The viscous dissipation rate per unit mass ε is given dimensionally by

$$\varepsilon = U^3/L \tag{11}$$

where L is a mixing length. The eddy viscosity ν_T is assumed to take one of the following forms

$$\nu_T = U^4/K\varepsilon \tag{12a}$$

$$\nu_T = U^4/K[\varepsilon + (3/2)(DU^2/Dt)] \tag{12b}$$

where $K = 5$ is another universal constant. The use of Eqs. (12a) and (11) leads to the well-known turbulent model of Prandtl,[14] and the use of Eq. (12b) leads to a jump structure at the turbulent interface. These two models are thus called Prandtl's model and the jump model, respectively.

The idea for modeling the eddy viscosity differently was brought forth by Lundgren[15] on the assumption that eddy viscosity governs the decay of anisotropy in turbulence. Relating a time scale for the decay of anistropy to the eddy viscosity by $\nu_T = U^2T$, Prandtl's model gives a decay time toward isotropy that is much shorter than the energy decay time, in contrast to experimental observations. The jump model was proposed to emphasize this point, giving infinite decay time for homogenous turbulence, yet is the same as Prandtl's model for steady turbulence. This distinction is not important in a fully developed turbulent flow, but is important in regions where rapid production exceeds the dissipation so as to increase the level of turbulence, such as for flow through a turbulent interface.

The viscous structure of a two-dimensional turbulent interface has been studied in Ref. 10 using the model equations. The resultant structures are quite different for the two models. While Prandtl's model has a smooth structure, the jump model leads to a sharp front. Jump discontinuities in mean velocity and turbulent energy across the front are obtained, which are related to the propagation speed of the front V by the jump relation

$$(V/U_j) = 0.36 + 0.072(u_j/V_j)^2 \tag{13}$$

Continuity of the Reynolds stress and turbulent energy at the interface gives the further relations

$$\tau_{xy} = Vu_j \qquad q_y = (\alpha/K)(U_j^4/V) \tag{14}$$

where x and y form a set of cartesian coordinates at the interface with y positive away from the turbulent region.

These relations provide boundary conditions for the analysis of fully developed turbulent flows. In Ref. 10, the structure of a Rayleigh shear layer was obtained by applying these conditions at a mean interface position. In this paper, the same conditions are applied to flows of plane jets and wakes. Because of the large fluctuations in the interface position, the second averaging is used to obtain the mean solutions comparable with the conventional measurements.

III. Turbulent Plane Wake

The structure of a two-dimensional turbulent wake of a circular cylinder with its axis normal to a uniform stream of velocity U_∞ is studied first. Assuming no external pressure gradient, the inviscid model equations with the boundary-layer approximation can be written as

$$(\partial u/\partial x) + (\partial v/\partial y) = 0 \tag{15}$$

$$u(\partial u/\partial x) + v(\partial u/\partial y) = (\partial/\partial y)[\nu_T(\partial u/\partial y)] \tag{16}$$

$$n\frac{U^3}{\bar{h}} + \frac{3}{2}\left(u\frac{\partial U^2}{\partial x} + v\frac{\partial U^2}{\partial y}\right) = \nu_T\left(\frac{\partial u}{\partial y}\right)^2 + \frac{\partial}{\partial y}\left(\frac{3}{2}\nu_T\frac{\partial U^2}{\partial y}\right) \tag{17}$$

Here x is positive in the downstream direction, and the mixing length L is assumed to be proportional to the width of the turbulent region $\bar{h}(x)$.

Experimental evidence[5] indicates that self-preservation of the flow structure exists far downstream where the freestream velocity is approached. Using the defect velocity $\tilde{u} = u - U_\infty$ and assuming that $\tilde{u} \ll U_\infty$, $v \ll U_\infty$, Eqs.(16) and (17) can be linearized to give

$$U_\infty(\partial\tilde{u}/\partial x) = (\partial/\partial y)[\nu_T(\partial\tilde{u}/\partial y)] \tag{16a}$$

$$n\frac{U^3}{\bar{h}} + \frac{3}{2}U_\infty\frac{\partial U^2}{\partial x} = \nu_T\left(\frac{\partial\tilde{u}}{\partial y}\right)^2 + \frac{\partial}{\partial y}\left(\frac{3}{2}\nu_T\frac{\partial U^2}{\partial y}\right) \tag{17a}$$

where the velocity component v no longer appears.

The boundary conditions for these equations are as follows: At the turbulent interface, $y = \bar{h}(x)$, there are two conditions which are either

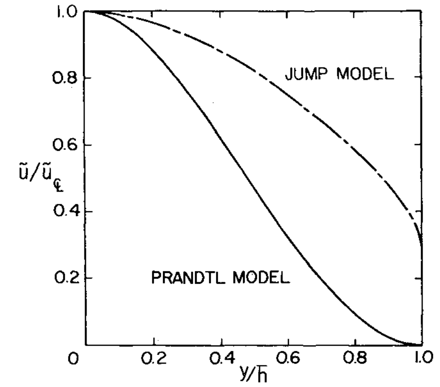

Fig. 1 Mean velocity distribution across the plane wake before averaging.

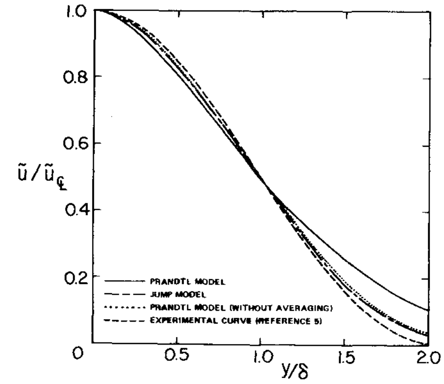

Fig. 3 Mean velocity distribution across the plane wake after averaging.

$$\tilde{u} = U^2 = 0 \tag{18}$$

for Prandtl's model, or the relations obtained from Eqs. (13 and 14) by eliminating V. At $y = 0$ there are two symmetry conditions

$$(\partial\tilde{u}/\partial y) = (\partial U^2/\partial y) = 0 \tag{19}$$

and an empirical condition from Townsend's data,[5]

$$(U/\tilde{u}) = 0.279 \tag{20}$$

used to determine the mixing length constant n. A conservation of integral momentum is used to determine the interface position $\bar{h}(x)$.

Introducing the similarity transformation

$$\tilde{u} = V(x)W(\xi) \qquad U^2 = V(x)^2\Theta(\xi)$$
$$v_T = V(x)\bar{h}(x)N(\xi) \qquad \xi = y/\bar{h}(x)$$

where $\bar{h} \propto (x)^{1/2}$ and $V = U_\infty d\bar{h}/dx$ are the mean interface position and entrainment velocity at the interface and after integrating Eq. (16a) once using the condition $\partial\tilde{u}/\partial y = 0$ at $y = 0$, the following system of equations is obtained

$$\begin{aligned} N(dW/d\xi) &= -\xi W \\ N(d\Theta/d\xi) &= -(2/3)Q \\ N(dQ/d\xi) &= \xi^2 W^2 - (n\Theta^{3/2} - 3\Theta)N - \xi Q \end{aligned} \tag{21}$$

Here Q is the nondimensional turbulent energy flux in the y-direction.

The eddy viscosity assumption and the boundary conditions are transformed into

$$N = \Theta^{1/2}/5n \tag{22a}$$

$$W(1) = \Theta(1) = 0 \tag{23a}$$

for Prandtl's model,

$$N = (\Theta^2/5 - \xi Q)/(n\Theta^{3/2} - 3\Theta) \tag{22b}$$

$$Q(1) = (1/5)\Theta(1)^2 \tag{23b}$$

$$\Theta(1)^{1/2} = 0.36\Theta(1) + 0.072W(1)^2 \tag{23c}$$

for the jump model, and

$$Q(0) = 0 \tag{24}$$

$$\Theta(0)^{1/2} = 0.279W(0) \tag{25}$$

for both models.

The boundary conditions at $\xi = 1$ imply that N tends to zero there, making it a singular point for the system of equations. To compute the solution near $\xi = 1$, an asymptotic series in terms of $\eta = 1 - \xi$ is obtained. For both models, two unknown constants are needed to define the series.

The differential equations are solved using the initial-value method of shooting. Solutions using $n = 2.42$ for Prandtl's model and $n = 2.07$ for the jump model satisfy the condition Eq. (25) at $\xi = 0$. The resulting distributions of mean velocity and turbulent energy are plotted in Figs. 1 and 2. The same curves after second averaging are plotted in Figs. 3 and 4. Experimental curves of Townsend[5] and the curves of Prandtl's model without second averaging are also plotted for comparison.

IV Turbulent Plane Jet

The governing equations for a self-preserving jet are given in Eqs. (15–17) of Sec. III. The boundary conditions for quiescent surroundings are as follows. At $y = \bar{h}(x)$, there are two interface conditions similar to those of the wake flow. At $y = 0$, there are two symmetry conditions for u, and U^2, and v must vanish there. An empirical relation from Bradbury[6] gives

$$(U/u) = 0.213 \quad \text{at} \quad y = 0 \tag{26}$$

Introducing similarity transformation

$$u = V(x)W(\xi) \qquad v = V(x)H(\xi) \qquad U^2 = V(x)^2\Theta(\xi)$$
$$v_T = V(x)\bar{h}(x)N(\xi) \qquad \xi = y/\bar{h}(x)$$

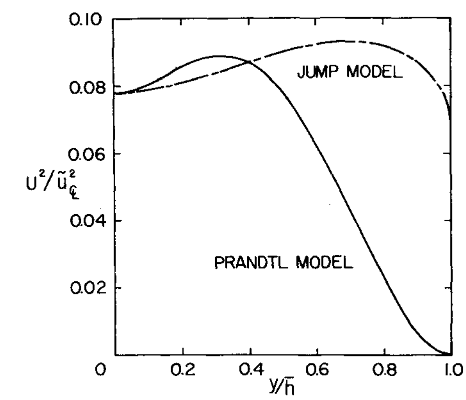

Fig. 2 Turbulent energy distribution across the plane wake before averaging.

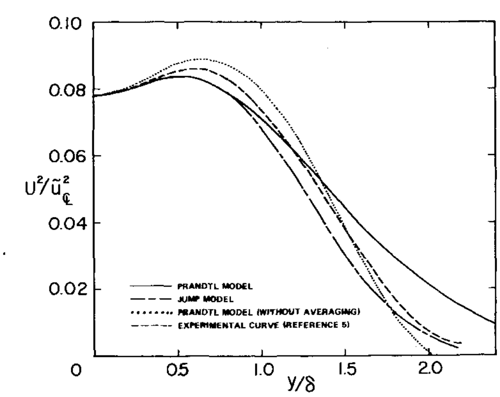

Fig. 4 Turbulent energy distribution across the plane wake after averaging.

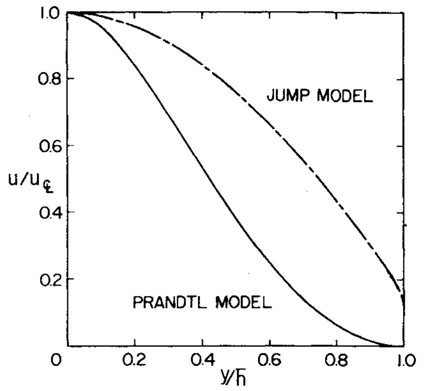

Fig. 5 Mean velocity distribution across the plane jet before averaging.

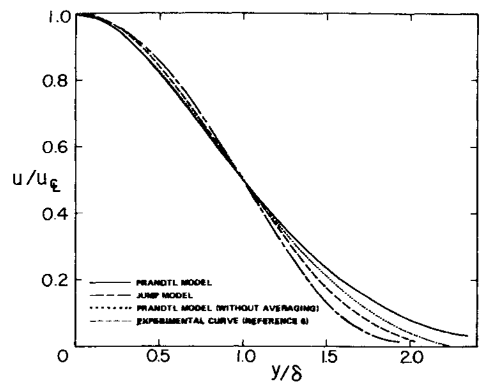

Fig. 7 Mean velocity distribution across the plane jet after averaging.

and after integrating the momentum Eq. (16) once using the condition $\partial u/\partial y = 0$ at $y = 0$, the following equations are obtained:

$$\begin{aligned} N(dW/d\xi) &= WE \\ (dE/d\xi) &= -(A/2)W \\ N(d\Theta/d\xi) &= -(2/3)Q \\ N(dQ/d\xi) &= W^2E^2 - [n\Theta^{3/2} - (3/2)AW\Theta]N + EQ \end{aligned} \tag{27}$$

Here

$$\bar{h} \propto x = Ax$$

is the mean turbulent interface position, and

$$V(x) \propto 1/(x)^{1/2}$$

is the entrainment velocity at the interface. The quantity

$$E = H - \xi AW$$

is the modified velocity component.

The eddy viscosity assumption and the boundary conditions become

$$N = \Theta^{1/2}/5n \tag{28a}$$

$$W(1) = \Theta(1) = 0 \tag{29a}$$

for Prandtl's model,

$$N = [(1/5)\Theta^2 + EQ]/[n\Theta^{3/2} - (3/2)AW\Theta] \tag{28b}$$

$$Q(1) = -(1/5)\Theta(1)^2/E(1) \tag{29b}$$

$$\Theta(1)^{1/2} = 0.36\Theta(1) - 0.072W(1)E(1) \tag{29c}$$

$$E(1) = -(1 + A^2)^{1/2} \tag{29d}$$

for the jump model, and

$$E(0) = Q(0) = 0 \tag{30}$$

$$\Theta(0)^{1/2} = 0.213W(0) \tag{31}$$

for both models.

The end point $\xi = 1$ is again a singular point, and asymptotic series in terms of $\eta = 1 - \xi$ are used to start integration. Numerical solutions using $n = 3.1$ for Prandtl's model and $n = 2.29$ for the jump model satisfy the condition (31).

Table 2 Numerical results for plane jets

Model	Values of constants		Angles of spread (deg.)	
	n	A	$\Theta_{1/2}$	Θ_0
Jump	2.29	0.1761	7.10	13.92
Prandtl's	3.10	0.2560	5.92	16.14
Prandtl's without averaging			6.16	14.67
Experimental data				
Bradbury[6]			6.22	
Forthmann[16]				15.29

Solution curves are plotted in Figs. 5 and 6, with the same curves after second averaging plotted in Figs. 7 and 8. Comparisons are made with the experimental curves of Bradbury.[6] The rate of spread of the jet is given from the constant A. Expressed in terms of the half angles θ_0 and $\theta_{1/2}$, they are listed in Table 2. Experimental data of Bradbury[6] and of Forthmann[16] are also listed for comparison.

V. Discussions and Conclusions

Good overall agreement with experiments has been achieved using the simple eddy viscosity formulations. The ability to predict the turbulent energy distributions is recognized. The discrepancies found here can be attributed to the neglect of the external potential flow, the simplified assumptions for the mixing length distribution and the approximate nature of the models.

The difference between the two models shows very clearly in the fine structure of turbulence. While Prandtl's model gives a smooth solution, the jump model has discontinuities at the turbulent interface. The largest jump occurs in the turbulent

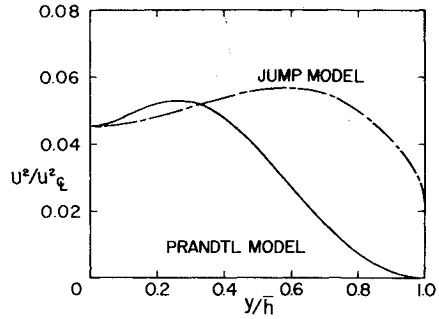

Fig. 6 Turbulent energy distribution across the plane jet before averaging.

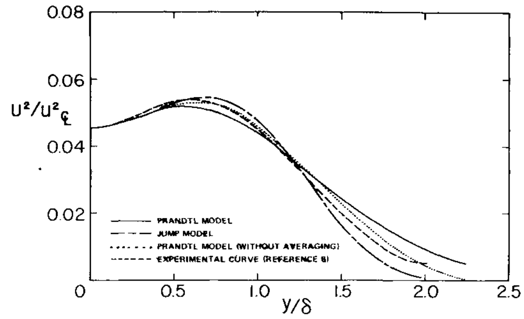

Fig. 8 Turbulent energy distribution across the plane jet after averaging.

energy distribution for the plane wake. This makes the distribution nearly uniform in the turbulent region, coinciding with Townsend's calculations.[9] The other distributions have smaller jumps, and thus are less uniformly distributed.

The relative jump values for the plane wake, $\tilde{u}_j/\tilde{u}_{℄} = 0.3$ and $U_j/\tilde{u}_{℄} = 0.264$, are found to be higher than the corresponding values for the plane jet, having $u_j/u_{℄} = 0.1$ and $U_j/u_{℄} = 0.148$. These values are in turn higher than those for the Rayleigh shear layer, found to be $u_j/u_\infty = 0.018$ and $U_j/u_\infty = 0.026$ in Ref. 10. This same sequence for the order of magnitude is also observed in the standard deviation $\sigma/\bar{h}$ of the turbulent interface distribution (Table 1). This correlation may be of physical significance. More work is needed to establish a definite relation.

The differences between the two models are largely reduced after the second averaging. Of all the resulting curves, Prandtl's model after the second averaging gives the most deviation from the experimental curves. The same curves without averaging give much better agreement. It is thus concluded that the second averaging process is not satisfactory with Prandtl's model under the present formulation. A more sophisticated form for the mixing length will probably give better results after the second averaging.

The assumption that the turbulent interface position is a single valued function of its coordinates is not valid at all times. There is experimental evidence[17] indicating that the interface is a multiple valued function of its coordinates for a nonnegligible fraction of time. However, close agreement between the probability distribution of the interface and the intermittency profile allows one to overlook the multivalued character when dealing with the global features of the flow. The second averaging process is thus valid only statistically.

The structure of the turbulent interface provides the key to the validity of the models. Conventional turbulent measurements give mean values irrespective of the interface position, so they cannot detect the jumps in the interface structure. The conditional sampling technique developed recently[11] gives mean values similar to the fine structures obtained here. Although jumps in mean vorticity have been found in some flows, no jump in mean velocity has been found yet. Based on information available currently, the same tentative conclusion is drawn as in Ref. 10 in favor of the simpler model of Prandtl's.

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